Attendance / Ice-Breaker

Which Moo Deng are you today?

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OFFICE HOURS CHANGE: now Wednesdays 10:30am-12:20pm in Huang 203

Announcements

OFFICE HOURS CHANGE: now Wednesdays 10:30am-12:20pm in Huang 203

- Midterm 1 on Tuesday (NO ACE SECTION)
- Covers material from lectures 0-5 and problem sets 1-2
- ACE Midterm resources:
  - Practice Exam: Saturday 10am-1pm, (likely 200-205)
    - Practice Exam 3 (with a few modifications), printed and in a lecture hall
  - Review Session: Sunday 10am-11:45am, LATHROP 298
    - 60-90 min concept review (recorded)
    - 15-45 min practice questions: proofs on Practice Exam 5
- Main Course resources:
  - Review Session: Sunday 2-3pm, STLC 115
- Resources: Academic Coaching, CTL Tutoring

Start thinking about Midterm 1

What can you do right now to prepare? **Make a study plan!**

- List all topics from lectures 0-5/psets 1-2
  - Decide what you are “confident” and “not confident” in
  - Decide what is “memorized” and “need to look up”
- Put your “need to look up”s on your sheet
- Spend time learning and practicing your “not confident” topics:
  - do extra ACE problems
  - re-write proofs from lecture or the psets without looking at the solutions
  - come to office hours or ask me questions on Slack

Keep this table open!

	To ***prove*** that this is true…	If you ***assume*** this is true…
$\forall x.\ A$	Have the reader pick an arbitrary x. We then prove A is true for that choice of x.	Initially, ***do nothing***. Once you find a z through other means, you can state it has property A.
$\exists x.\ A$	Find an x where A is true. Then prove that A is true for that specific choice of x.	Introduce a variable x into your proof that has property A.
$A \rightarrow B$	Assume A is true, then prove B is true.	Initially, ***do nothing***. Once you know A is true, you can conclude B is also true.
$A \land B$	Prove A. Then prove B.	Assume A. Then assume B.
$A \lor B$	Either prove $\neg A \rightarrow B$ or prove $\neg B \rightarrow A$. *(Why does this work?)*	Consider two cases. Case 1: A is true. Case 2: B is true.
$A \leftrightarrow B$	Prove $A \rightarrow B$ and $B \rightarrow A$.	Assume $A \rightarrow B$ and $B \rightarrow A$.
$\neg A$	Simplify the negation, then consult this table on the result.	Simplify the negation, then consult this table on the result.

Functions

- $f : A \to B$ notates that f is a function with **domain** A and **codomain** B
- Rules of functions:
  - $\forall a \in A.\ \exists b \in B.\ f(a) = b$
    - *f* can only be applied to elements of its domain
    - For any *x* in the domain, *f*(*x*) is an element of the codomain
  - $\forall a_1 \in A.\ \forall a_2 \in A.\ (a_1 = a_2 \to f(a_1) = f(a_2))$
    - Equal inputs produce equal outputs
- Piecewise functions
  - For every x in the domain, at least one rule has to apply
  - All applicable rules should give the same result
  - Example:

$$f(n) = \begin{cases} k & \text{if } \exists k \in \mathbb{N}.\, n = 2k \\ -(k+1) & \text{if } \exists k \in \mathbb{N}.\, n = 2k+1 \end{cases}$$

Injectivity

$$\forall a_1 \in A. \forall a_2 \in A. (a_1 \neq a_2 \rightarrow f(a_1) \neq f(a_2))$$

How to prove injectivity 1:

- Pick a_1 and a_2 in A where $a_1 \neq a_2$
- We will show that $f(a_1) \neq f(a_2)$

Read more: Proof Template

Injectivity

$$\forall a_1 \in A. \forall a_2 \in A. (f(a_1) = f(a_2) \rightarrow a_1 = a_2)$$

How to prove injectivity 2:

- Pick a_1 and a_2 in A where $f(a_1) = f(a_2)$
- We will show that $a_1 = a_2$

Read more: Proof Template

Surjectivity

$$\forall b \in B.\exists a \in A.(f(a) = b)$$

How to prove surjectivity:

- Pick b in B
- Show that there exists a in A where f(a) = b
    - To do so, give a value for a
    - Then explain why f(a) = b

Read more: Proof Template

Summary: Special types of functions

$f : A \to B$ is **injective** (one-to-one) if either of these equivalent statements is true:

$$\forall x_1 \in A.\ \forall x_2 \in A.\ (x_1 \neq x_2 \to f(x_1) \neq f(x_2))$$

$$\forall x_1 \in A.\ \forall x_2 \in A.\ (f(x_1) = f(x_2) \to x_1 = x_2)$$

An injective function associates at most one element of the domain with each element of the codomain.

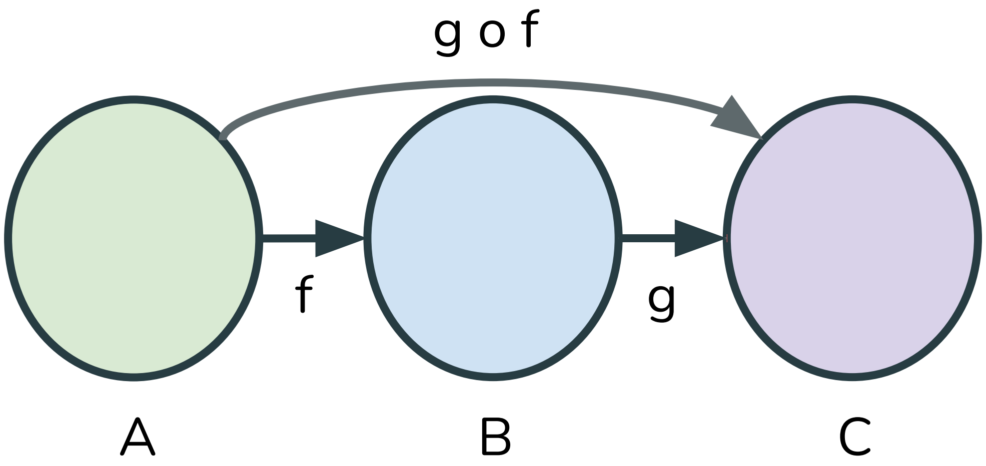
$f : A \to B$ is **surjective** (onto) if it has this property:

$$\forall b \in B.\ \exists a \in A.\ f(a) = b$$

A surjective function associates at least one element of the domain with each element of the codomain.

$f : A \to B$ is **bijective** if it is both injective and surjective.

Function Composition



Tip: When substituting into definitions, treat (g o f) as one unit / a function name.
When expanding definitions or applying a function, then you can substitute g(f(x)) for (g o f)(x)

	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$
$S = T$	$S \subseteq T \land T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
$x \in A \cap B$	$x \in A \land x \in B$	Assume $x \in A$. Then assume $x \in B$.	Prove $x \in A$. Also prove $x \in B$.
$x \in A \cup B$	$x \in A \lor x \in B$	Consider two cases: Case 1: $x \in A$. Case 2: $x \in B$.	Either prove $x \in A$ or prove $x \in B$.
$X \in \wp(A)$	$X \subseteq A$.	Assume $X \subseteq A$.	Prove $X \subseteq A$.
$x \in \{ y \mid P(y) \}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

Problem 4 Walkthrough: Power Set Subsets

To prove one set is a subset of another set,

- Have the reader pick an arbitrary element from the **first set**
- Show it is an element in the **second set**

Memorize this! It will be used for almost every subset proof!

Set equality proofs are two subset proofs

Problem 4 Walkthrough: Power Set Subsets

Assume:

Want to show:

$$\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Want to show:

$$\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

Want to show:

S is an element of $\wp(A \cap B)$

Start every subset proof like this!

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

Want to show:

S is an element of $\wp(A \cap B)$

We have **definitions** on each side, so let's expand them.

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

$\rightarrow$ S is an element in $\wp(A)$

$\rightarrow$ S is an element in $\wp(B)$

Want to show:

S is an element of $\wp(A \cap B)$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

$\rightarrow$ S is an element in $\wp(A)$

$\rightarrow$ S is a subset of A

$\rightarrow$ S is an element in $\wp(B)$

$\rightarrow$ S is a subset of B

Want to show:

S is an element of $\wp(A \cap B)$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

$\rightarrow$ S is an element in $\wp(A)$

$\rightarrow$ S is a subset of A

$\rightarrow$ S is an element in $\wp(B)$

$\rightarrow$ S is a subset of B

Want to show:

S is an element of $\wp(A \cap B)$

S $\subseteq$ $A \cap B$

It's another subset proof!

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

$\rightarrow$ S is an element in $\wp(A)$

$\rightarrow$ S is a subset of A

$\rightarrow$ S is an element in $\wp(B)$

$\rightarrow$ S is a subset of B

Pick x from S

Want to show:

S is an element of $\wp(A \cap B)$

S $\subseteq$ $A \cap B$

x is an element of $A \cap B$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

$\rightarrow$ S is an element of $\wp(A)$

$\rightarrow$ **S is a subset of A**

$\rightarrow$ S is an element of $\wp(B)$

$\rightarrow$ S is a subset of B

Pick **x from S**

Want to show:

S is an element of $\wp(A \cap B)$

S $\subseteq$

When we **assume** subset relationships, we get information about **elements of the left-hand set**

x is an element of $A \cap B$

	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$
$S = T$	$S \subseteq T \land T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
$x \in A \cap B$	$x \in A \land x \in B$	Assume $x \in A$. Then assume $x \in B$.	Prove $x \in A$. Also prove $x \in B$.
$x \in A \cup B$	$x \in A \lor x \in B$	Consider two cases: Case 1: $x \in A$. Case 2: $x \in B$.	Either prove $x \in A$ or prove $x \in B$.
$X \in \wp(A)$	$X \subseteq A$.	Assume $X \subseteq A$.	Prove $X \subseteq A$.
$x \in \{ y \mid P(y) \}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.